unit 8 packet

Quadratic Functions

This Unit 8 Packet contains 6 lessons and some additional practice:
8-1 Exploring Quadratic Graphs
8-2 Quadratic Functions (Part # 1)
8-2 Quadratic Functions (Part # 2)
Practice: 8-2 Quadratic Functions Worksheet
8-3 Finding x - Intercepts of Quadratic Functions (Part # 1)
Practice: 8-3 Finding x - Intercepts Worksheet #1
8-4 Vertex Form and Transformations (Part 1)
8-4 Vertex Form Worksheet # 1
8-4 Vertex Form and Transformations (Part 2)
8-4 Vertex Form Worksheet # 2

- Quadratics notes to understand wth is going on
  - quadratics notes
  - The quadratic formula:
  - how to find x and y intercepts
  - when i know the y value of the vertex, how do i find the x value?
- 8-2 Quadratic Functions (Part # 1)
  - Example 1
  - Example 2 - Vertex Formula
  - Example 3 + up/down test
  - Example 4 Graph f(x) = x^2 -2x - 8
  - Example 5: Graph y = x^2 + 2x + 3
  - Example 6: Graph y = 2x^2 - 8x
  - Example 7: h = 16t^2 + 72t + 520
- quadratics vocabulary

- Quadratics Vocabulary

Quadratics notes to understand wth is going on

quadratics notes

- Quadratic equation standard form: $ax^2 + bx + c = 0$ This is the standard form for any quadratic equation, where a, b, and c are constants. It can be used to find the roots or zeros of the equation by using the quadratic formula.

- Quadratic formula: $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$ This formula is used to find the roots (x-intercepts) of a quadratic equation in standard form. It gives the two possible values for x when the equation is equal to zero.

- Axis of symmetry: $x = -b / 2a$ This formula is used to find the vertical line that divides the parabola into two symmetrical halves. It can help in graphing the quadratic function and finding the vertex.

- Vertex form: $y = a(x - h)^2 + k$ This form is useful for easily identifying the vertex of a quadratic equation, where (h, k) is the vertex point. It can be used to graph the parabola or find the maximum/minimum value.

- Completing the square: $x^2 + bx + c = a(x - h)^2 + k$ This method is used to convert a quadratic equation from standard form to vertex form. It involves adding and subtracting a constant term to make the left side a perfect square trinomial.

- Discriminant: $\Delta = b^2 - 4ac$ The discriminant is used to determine the nature of the roots of a quadratic equation. It can help predict whether the equation has real or complex roots and how many distinct solutions it has.

- Factored form: $y = a(x - p)(x - q)$ This form is useful for quickly identifying the roots of a quadratic equation, where p and q are the x-intercepts. It can also be used to find the factors of the quadratic equation.

- Parabola focus: $F(h, k + 1/4a)$ The focus is a point that defines the geometric property of a parabola. It can be used to find the directrix and derive the equation of a parabola from its geometric definition.

- Parabola directrix: $y = k - 1/4a$ The directrix is a horizontal line that is equidistant from the focus and vertex of the parabola. It can be used in the geometric definition of a parabola and to find its equation from given points.

The quadratic formula:

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$$

The formula gives you two possible results for the values of "x" that satisfy the quadratic equation $y = ax^2 + bx + c$.

The two possible values of "x" correspond to the x-coordinates of the points where the parabolic curve intersects the x-axis, also known as the x-intercepts or roots of the quadratic equation.

If the discriminant ($b^2 - 4ac$) is positive, then the quadratic equation has two real roots, and the parabolic curve intersects the x-axis at two distinct points. In this case, you will get two different values of "x" when you use the quadratic formula.

If the discriminant is zero, then the quadratic equation has one real root with a multiplicity of two, and the parabolic curve touches the x-axis at exactly one point. In this case, you will get the same value of "x" twice when you use the quadratic formula.

If the discriminant is negative, then the quadratic equation has two complex roots, and the parabolic curve does not intersect the x-axis. In this case, you will get two different complex values of "x" when you use the quadratic formula.

In summary, the two possible results from the quadratic formula correspond to the x-coordinates of the points where the parabolic curve intersects the x-axis, and the number and nature of these points depend on the value of the discriminant.

how to find x and y intercepts

- Find the x-intercept by plugging in 0 for y.
- Find the y-intercept by plugging in 0 for x

when i know the y value of the vertex, how do i find the x value?

If you know the y-coordinate of the vertex of a quadratic function, you can use the vertex form of the function to find the x-coordinate of the vertex.

The vertex form of a quadratic function is:

$y = a(x - h)^2 + k$

where (h, k) is the vertex of the parabola.

If you know the y-coordinate of the vertex, which is k in the vertex form equation, you can substitute it into the equation to get:

$y = a(x - h)^2 + k$

Simplifying this equation, we get:

$y - k = a(x - h)^2$

Dividing both sides by "a", we get:

$(y - k) / a = (x - h)^2$

Taking the square root of both sides, we get:

$\sqrt{(y - k) / a} = x - h$

Adding "h" to both sides, we get:

$x = h \pm \sqrt{(y - k) / a}$

So, to find the x-coordinate of the vertex, you can use the formula:

$x = h \pm \sqrt{(y - k) / a}$

where "h" is the x-coordinate of the vertex, "k" is the y-coordinate of the vertex, and "a" is the coefficient of the x^2 term in the quadratic function.

8-2 Quadratic Functions
(Part # 1)

Example 1

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- A) The vertex is (4,3)
- B) The vertex is (-3,-3)

Example 2 - Vertex Formula

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a) y = 2x^2+4x

1. we get the a, b, c in the formula y = ax^2 + bx + c
    - a = 2
    - b = 4
    - c = 0
2. then we plug in a, b into the formula
    - x = -b / (2a)
    - x = -4 / (2*2) = -4/4 = -1

- so the axis of symmetry is x = -1
- to find the vertex
- so the known vertex data is (-1, y)
- to find y for the vertex we need to plug -1 into the formula y = 2x^2+4x
- y = 2*-1^2+4*-1 = -6

- so the vertex is (-1, -6)

  -=-

B) y = -x^2 + 4x - 5

1. we get the a, b, c in the formula y = ax^2 + bx + c
    - a = -1
    - b = 4
    - c = -5
2. then we plug in a, b into the formula
    - x = -b / (2a)
    - x = -4 / (2*-1) = -4/-2 = 2

- so the axis of symmetry is x = 2
- to find the vertex
- so the known vertex data is (2, y)
- to find y for the vertex we need to plug -1 into the formula y = 2x^2+4x
- y = -2^2 + 4 * 2 - 5 = -1

- so the vertex is (2, -1)

Example 3 + up/down test

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up/down test

- opens upwards because a is Positive
- opens downward because a is Negative

Example 3

1. a) y= x^2 + 3x + 4
   - opens upwards because a is Positive
2. b) y = -3x^2 + 5x
   - opens downward because a is Negative
3. c) y = 2x - x^2 + 6
   - opens downward because a is Negative

Example 4 Graph f(x) = x^2 -2x - 8

Steps to Graph ax^2 + bx + c

- Find the vertex and the axis of symmetry. Sketch these in.
- Find the x-intercept by plugging in 0 for y.
- Find the y-intercept by plugging in 0 for x.
- Reflect your points across the axis of symmetry and connect your dots with a smooth U-shaped (not V-shaped) curve.

Graph f(x) = x^2 -2x - 8

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for now, I'm just gonna type my work and figure out what to do next

1. find the line of symmetry -
    1. a = 1, b = -2, c = -8
2. use this to find the vertex
    1. x = (b/2a)
    2. x = -(-2) / 2(1) = 1
3. since we know that the along the x axis at 1 will be the vertex we replace x with 1 in the original formula
    1. x=1
    2. y = x^2 -2x - 8
    3. y = 1^2 + -2 * 1 - 8 = 1- 2 - 8 = -9
    4. y = -9
4. the vertex is (1, -9)
5. since the vertex is -1,-9 we know that x=1 is the axis of symmetry
6. finding the y-intercept is the easiest to start with because we just replace x with 0
7. x = 0 | y = x^2 -2x - 8
8. y = 0 - 8 = -8
9. y-intercept = (0,-8)
10. so so To find the x-intercepts, you can set y equal to zero and solve for x:
11. y = 0 | x = (-b ± sqrt(b^2 - 4ac)) / 2a

    1. x = (-(-2) ± sqrt((-2)^2 - 4(1)(-8))) / 2(1)
       x = (2 ± sqrt(4 + 32)) / 2
       x = (2 ± sqrt(36)) / 2
       x = (2 ± 6) / 2
       x = 8 / 2 or x = -4/ 2
       x = 4 or x = -2
       sooooo (-2,0) & (4,0)
12. so since we know 3 y axis points on the graph and the axis of symmetry we can get another point without doing much work
    1. symmetry line = x = 1,
    2. calc'd x-intercept 0,-8
        1. the symmetry line is 1 and the known point is 0 since 1-0 = 1 we can add that to the x coordinate of y and keep the same y coordinate to get the mirrored point making another point on the graph (2,-8)
    3. since we need one more point for the graph we can choose say x=3, | x^2 -2x - 8
        1. y = 3^2 -3*2 - 8 = -5
            1. soooo the new point is (3,-5) if we mirror that along 1,-9 we get (-1, -5) because 3 is 2 more than 1, and 2 less than 1 is -1. we also keep the same y coordinate
    4. so all points are:
        1. (1, -9)
        2. (0,-8)
        3. (2, -8)
        4. (3,-5)
        5. (-1,-5)

Example 5: Graph y = x^2 + 2x + 3

Find the vertex and the axis of symmetry. Sketch these in.
• Find the x-intercept by plugging in 0 for y.
• Find the y-intercept by plugging in 0 for x.
• Reflect your points across the axis of symmetry and connect your dots with a smooth U-shaped (not V-shaped) curve.

fix the fallowing

- a = 1, b = 2, c = 3
- x^2 + 2x + 3

1. find the line of symmetry -
   1. x = (b/2a)
   2. x = -(2) / 2(1) =1
2. use this to find the vertex
3. since we know that the along the x axis at -1 will be the vertex we replace x with 1 in the original formula
   1. x=1
   2. y = x^2 -2x - 8
   3. y = 1^2 + -2 * 1 - 8 = 1- 2 - 8 = -9
   4. y = -9
4. the vertex is (1, -9)
5. since the vertex is -1,-9 we know that x=1 is the axis of symmetry
6. finding the y-intercept is the easiest to start with because we just replace x with 0
7. x = 0 | y = x^2 -2x - 8
8. y = 0 - 8 = -8
9. y-intercept = (0,-8)
10. so so To find the x-intercepts, you can set y equal to zero and solve for x:
11. y = 0 | x = (-b ± sqrt(b^2 - 4ac)) / 2a
    1. x = (-(-2) ± sqrt((-2)^2 - 4(1)(-8))) / 2(1)
       x = (2 ± sqrt(4 + 32)) / 2
       x = (2 ± sqrt(36)) / 2
       x = (2 ± 6) / 2

x = 8 / 2 or x = -4/ 2

x = 4 or x = -2

sooooo (-2,0) & (4,0)

12. so since we know 3 y axis points on the graph and the axis of symmetry we can get another point without doing much work
    1. symmetry line = x = 1,
    2. calc'd x-intercept 0,-8
        1. the symmetry line is 1 and the known point is 0 since 1-0 = 1 we can add that to the x coordinate of y and keep the same y coordinate to get the mirrored point making another point on the graph (2,-8)
    3. since we need one more point for the graph we can choose say x=3, | x^2 -2x - 8
        1. y = 3^2 -3*2 - 8 = -5
            1. soooo the new point is (3,-5) if we mirror that along 1,-9 we get (-1, -5) because 3 is 2 more than 1, and 2 less than 1 is -1. we also keep the same y coordinate
    4. so all points are:
        1. (1, -9)
        2. (0,-8)
        3. (2, -8)
        4. (3,-5)
        5. (-1,-5)

Example 6: Graph $y = 2x^2 - 8x$

• Find the vertex and the axis of symmetry. Sketch these in.
• Find the x-intercept by plugging in 0 for y.
• Find the y-intercept by plugging in 0 for x.
• Reflect your points across the axis of symmetry and connect your dots with a smooth U-shaped (not V-shaped) curve.

Example 7: $h = 16t^2 + 72t + 520$

Suppose a particular “star” is projected from a firework at a starting height of 520 feet with an initial upward velocity of 72 ft/sec.

The equation:

$h = 16t^2 + 72t + 520$

gives the star’s height h in feet at time t in seconds.

a) How long will it take for the star to reach its maximum height?

b) What is the maximum height?

quadratics vocabulary

Quadratics Vocabulary

- Vertex
    - The vertex is the highest or lowest point on the curve.
    - The vertex is the point (x, y) where x = -(b/2a).
    - Naru calls it the apex.
    - We then use this x-value in the equation to find the y-value of the vertex.
- Vertex Formula:
    - The graph of y = ax^2 + bx + c has the line x = -(b/2a) as its axis of symmetry.
    - The x-coordinate of the vertex is x = -(b/2a)
    - You can find the y by plugging the x-cordonate into your equation.
- axis of symmetry
    - the vertical line that splits the parabola down the middle.
    - The axis of symetry uses the same formula as the vertex formula to find the x-cordonate: x = -(b/2a)
- Up/ Down Test